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ORIGINAL PAPER

The impact of inventory sharing on the bullwhip effect in decentralized inventory systems

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Abstract The paper derives the impact of inventory sharing policy on the bullwhip effect in two-stage supply chains with two independent suppliers and two integrated retailers. There exists an inventory sharing policy between two retailers. Under inventory sharing policy, when demand in one retailer exceeds its inventory, this retailer can ask for a product sharing volume from the other in order to satisfy customer demand. With certain assumptions, the bullwhip effect is quantified in both cases, with inventory sharing policy and without inventory sharing policy. We found that inventory sharing has significant impact on the bullwhip effect in the supply system. However, inventory sharing policy does not synchronously reduce or increase the bullwhip effect in both suppliers in the same period. A numerical example is given to illustrate the study model.

Keywords Inventory sharing policy · Bullwhip effect · Supply chain management · Decentralized inventory · Order lead time

1 Introduction

The information about customer demand is varied through the levels of the chain due to many factors (e.g., inventory policy, forecasting method, order lead time, etc.). In two-stage supply chain, retailers are the parties who receive customers' demands directly. To satisfy customers' service, usually customer demand is estimated by using forecasting techniques before placing order to supplier. The lacking of information leads to fluctuation orders from all levels of the chain in term of volumes. The fluctuation of customer demand through the chain is well known as the bullwhip effect [3, 19].

In Fig. 1, we can see the fluctuation of customers' demands through different facility epochs in a four-stage supply chain with single manufacturer, distributor, wholesaler, and retailer. We may notice that demands fluctuation increases from lower levels to higher levels in the chain. The reasons can be explained as follows: the retailer has directly customer information. Retailer will use this information to estimate actual demands. To maintain desired service level, retailer needs to hold a certain inventory in the warehouse. That leads to the wholesaler will receive higher original orders from the retailer. Similarly, wholesaler receives customer information from the retailer and places an order to his supplier, the distributor. To determine the order quantities from retailer, the wholesaler must forecast customer demand. Unfortunately, the wholesaler does not have access to the customer actual information; so that they must use the information from the retailer to perform his forecasting. Therefore, the variation of customer demand increases from lower epoch to higher epoch in the chain.

In the supply chain system applying inventory sharing policy, distribution centers, wholesalers, and retailers are collaborated by sharing product in case of emergency such that stock out or demand exceeds inventory. Separated inventory of parties in the same levels are virtually combined. If one party is in stock out stage, its demands can be fulfilled by available inventory in the other retailers.

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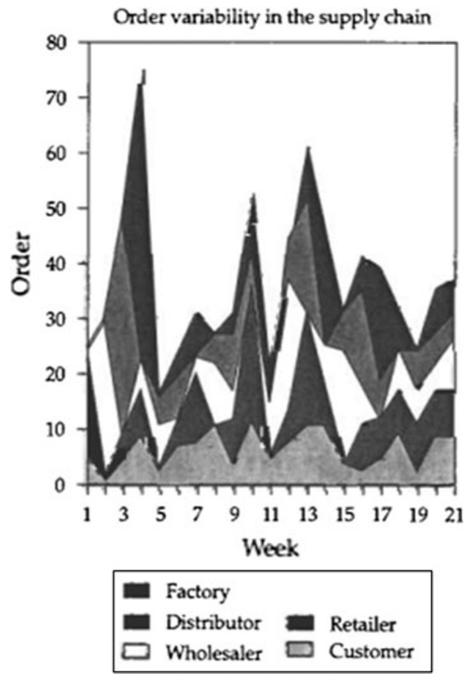

Fig. 1 Variation of customer demand in a supply chain system

This cooperation especially profits integrated supply chain system (the chain in which manufacturers and retailers are dependent). For a supply chain with independent facilities, inventory sharing policy sounds like a game changer and this policy would be more efficient in the centralized information system.

Inventory sharing can be profited for whole supply chain system including: manufacturers, distributors, retailers, and customers. Manufacturers can improve brand reputation, increase manufacturing efficiency, and reduce unwanted inventory. For distributors/retailers, inventory sharing reduces the number of lost orders and backorders, provide a new outlet for slow moving inventory, and increase possibility for incremental revenue (reduce inventory while remaining service level). Finally, customers also profit from inventory sharing in which it is an efficient strategy that improves product availability and reduces delivering time.

However, inventory sharing policy does not always guarantee making benefit for retailers. In case of transhipment, reallocation, and penalty costs are too high, retailers will try to reduce the probability of stock out by increasing inventory level. In addition, if the ordered lead time and order duration period time from suppliers to retailers is short, retailers may prefer waiting for new delivering packages instead of receiving sharing packages from other retailers. Therefore, the efficiency of inventory sharing strategy depends on geography factors, features of system, products, and services.

2 Literature review

The studies about bullwhip effect have been utilized for several decades. The earliest paper studied on this area is conducted by Forrester [8]. In this paper, the authors the first time pointed out the effects of information variation on production decision. This concept has been noticed and become foundation for research in the field of demand variation as we call "the bullwhip effect."

Main objectives of previous papers mainly focus on three aspects: (a) demonstrating the existence of the bullwhip effect, (b) identifying possible causes of the bullwhip effect, and (c) developing strategy to reduce the impact of the bullwhip effect [3]. Following the stream, our paper can be classified into the second areas, that is, identifying possible causes of the bullwhip effect. This researched area has been studying by many researchers. For instance, Lee et al. [20] pointed out in their papers four major causes of the bullwhip effect including: demand forecast updating, order batching, price fluctuation, rationing and shortage gambling. They also presented several methods to counteract the bullwhip effect such as integrating new information systems, defining new organizational relationships, and implementing new incentive and measurement systems.

The effect of forecasting methods on the bullwhip effect is also the main topic in many studies. Chen et al. [3] studied the impact of different forecasting methods on the bullwhip effect in a two-stage supply chain. They concluded that exponential smoothing forecasting method gives higher bullwhip effect than moving average forecasting method. In the same sense, Graves [25], Xu et al. [19] and Zhang [30] presented the effect of demand forecasts on the bullwhip effect in a two-stage supply chain system with integrated moving average demand process. Furthermore, Sun and Ren [13], Zhang [30] studied the impact of different forecasting methods such as MA, ES, EWMA, MMSE, and suggested the forecast method which can mitigate the bullwhip effect.

The impact of order lead time was presented in the papers of Graves [25], Chen et al. [9], Zhang [30], Lee et al. [20]. Luong and Phien [11] proved that the bullwhip effect can be decreased by reducing lead time. However, Duc et al. [28] showed that reducing lead time does not always reducing the bullwhip effect. He showed that in some special cases; for example, in a two-stage supply chain with a pre-specified ARMA demand process, increasing the lead time may help to reduce the bullwhip effect. For more detail, we refer to [28]. When considering the impact of lead time on the bullwhip effect, the lead time can be deterministic or stochastic. Results in those papers mentioned above are in case of deterministic lead time, whereas in practical lead time usually behaves as a stochastic process.

Chaffied [4] and So and Zheng [18] used simulation approaches to demonstrate the impact of lead time variation and information sharing on the customer demand fluctuation in a supply chain. Other results in this area are contented in [15, 16, 28].

Li et al. [31] analyzed the impact of demand substitution on the bullwhip effect in a two-stage supply chain with a singer supplier, singer retailer, and two types of products A and B such that a certain fraction product A can be used to substitute product B. They showed the relation between the bullwhip effect and the forecasting method, lead time, demand process, and the product substitution. The impact of demand substitution has been noticed and investigated by some previous authors [1, 2, 5, 21, 22, 24].

Inventory sharing has become important perception in some supply chain models. For example, in the supply chain with singer supplier and multi-independent retailers, the delivering time from suppliers to retailers' warehouses take a long time because of the long distance between suppliers and retailers, whereas retailers may located very closed in one area. There is probability that inventory of one retailer exceeds customer demand, while others are in stock out state. In this situation, stock out can be satisfied by transferring products among retailers through inventory sharing policy.

The concept of third-party warehouse is created aiming at identifying inventory policy and customer information. It may help to decrease total inventory at retailers. In this area, Duc et al. [29] studied the effect of third-party warehouse on the bullwhip effect. They found that third-party warehouse does not always reducing the bullwhip effect. They also stated the conditions in which the utilization of third-party warehouse decreases the bullwhip effect in supply chain.

Rudi [23] studied on the relation among following factors: inventory sharing, transhipment cost, and inventory orders in the supply chain with one supplier and two local retailers. They pointed out that inventory sharing and transhipment costs have significant effects on inventory order at each retailer. A case of study was conducted in which a two-stage supply chain with singer supplier, Bosch based in Germany, and five retailers based in Norway. In this supply chain system, the ordered delivering time from Germany to Norway takes about 3 weeks, while transhipment time within Norway is insignificant. This supply system is more similar with the supply chain model in our paper.

Zhao [14] presented an optimal inventory policy for each dealer in decentralized dealer networks. They concluded that (1) inventory sharing has big impact on the level of inventory by the independent dealer, (2) increasing inventory sharing leads to decreased dealers' rationing level rather than increasing their based stock level, (3) a smaller level of incentive for inventory sharing may be sufficient to achieve the benefit of full inventory sharing policy, (4) the benefit of inventory sharing increase the system utilization, and (5) customer service may improve significantly with inventory sharing.

Recently, Kutanoglu [7] considered a model to allocated stock level in the warehouse in a service part logistic network. The network includes one supplier with the infinitive warehouse capacity and a number of local warehouses. Each local warehouse has independent based stock policy. Moreover, local warehouses share their inventory as a way to increase service levels. They concluded that inventory sharing can reduce based stock levels and total system cost.

As mention above, the principle of the bullwhip effect is the variation of customer demand through the chain. Meanwhile, inventory sharing has significant impact on inventory levels and order quantities as well. That results in the change in customer information. Previous researches mainly focus on various areas of the bullwhip effect and inventory sharing. However, there are no works directly focusing on the impact of inventory sharing on the bullwhip effect yet. In this paper, we will derive this issue and examine the impact of inventory sharing policy on the bullwhip effect. The remaining of the paper is organized as follows: Sect. 3 describes the problem and develops mathematical formulation; Sect. 4 gives a numerical example, results, and discussion. Conclusions and recommendations for further study is the content of the last section.

3 Model development

3.1 Notations and assumptions

The following notations are used in this paper

- t — order period number index;
- i — retailer and corresponding supplier index, $i = 1, 2$;
- λ — percentage of product that one retailer may share to other retailers in the period t;
- L_i — the order lead time for retailer i, $i = 1, 2$;
- p — the number of demand observation periods used in the moving average forecast;
- μ_i — average demand of product at retailer i in the autoregressive demand model;
- $\varepsilon_{t,i}$ — forecast error for product at retailer i during time period t;
- ρ_i — the autocorrelation coefficient of the autoregressive model of product at retailer i; $|\rho_1| < 1, |\rho_2| < 1$
- $D_{t,i}$ — product demand at retailer i during time period t;

$D_{t,i}^{L_i}$ lead time demand for product at retailer i;
$\widehat{D}_{t,i}^{L_i}$ the forecast of the lead time demand product at retailer i;
z_i normal z-score determined by the desire service level;
$\sigma_{t,i}^{L_i}$ standard deviation of forecast error of lead time demand for product at retailer i;
$y_{t,i}$ order-up-to level inventory for retailer i at the beginning of time period t;
$q_{t,i}$ order quantity product for retailer i at the beginning of time period t;
B_i the bullwhip effect at supplier i;
$C_{L,\rho}$ a constant function of $L$and ρ

The proposal model is studying under following assumptions

As1 Order lead time of each retailer is smaller than duration time of one period $(L_i < \sigma(t), i = 1, 2)$.
As2 Time period index t and duration of order period are equal for both retailers
As3 In one order period, only one retailer agree share inventory, while the other will receive this amount of inventory
As4 Inventory sharing percentage is varying from period to period
As5 In every period, order from each retailer is positive

3.2 Problem description

In this paper, we consider a two-stage supply chain with two suppliers and two retailers with separate markets (each retailer has their own customer). A single product is delivered from suppliers to retailers following discrete time period.

The relationship among the parties in the chain is represented in Fig. 2. That is, whenever one retailer is in stock out state, they can ask for transhipment from the other retailer. Whether transhipment is made depending on predetermine conditions stated in the sharing policy. The process of satisfying customer demand with inventory sharing policy is illustrated in Fig. 3.

We assume that order lead times for both retailers (time from placing an order until receiving the order) are deterministic. We support that both retailers use the same forecasting method (MA) and inventory policy (order-up-to level). Since inventory sharing policy could affect order quantities that a retailer placing on its supplier, we will consider the impact of inventory sharing on the bullwhip effect of each retailer. The bullwhip effect can be determined by identifying the ratio of variance of retailer's order to the supplier to the variance of customer's demand to the retailers $\frac{\text{var}(q_i)}{\text{var}(D_i)}$, in which D_i is customer demand forecast, q_i is order quantity from retailer i placed at supplier i, $i = 1, 2$.

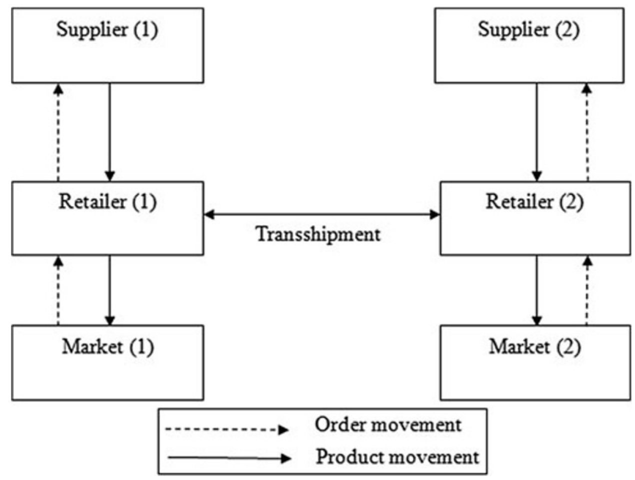

Fig. 2 Relationship among parties in the chain

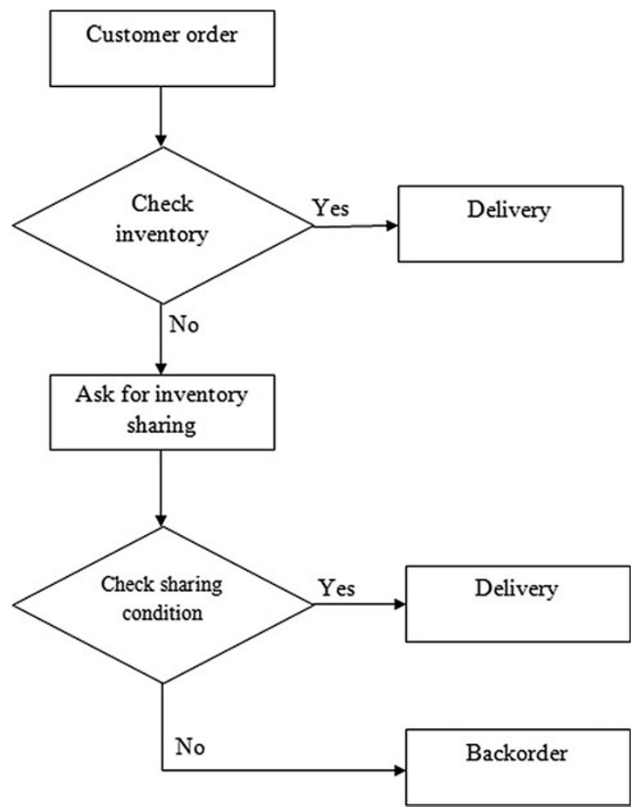

Fig. 3 Progress of satisfying customer demand

3.3 Mathematical formulation

Support at the beginning of order period t, both retailer (1) and retailer (2) estimate customer demand and place an order to its supplier. The order cost is negligible. From Zhang [30] and Gilbert [10], end customer demand can be modeled by an autoregressive (AR). That means end customer demand at each retailer during time period t can be determined as follows:

$$D_{t,1} = \mu_1 + \rho_1 D_{t-1,1} + \varepsilon_{t,1},\\ D_{t,2} = \mu_2 + \rho_2 D_{t-1,2} + \varepsilon_{t,2}. \tag{1}$$

From (1), we have (See "Appendix" for details)

$$E(D_{t,1}) = \frac{\mu_1}{1-\rho_1}; \quad \mathrm{var}(D_{t,1}) = \frac{\sigma_1^2}{1-\rho_1^2},\\ E(D_{t,2}) = \frac{\mu_2}{1-\rho_2}; \quad \mathrm{var}(D_{t,2}) = \frac{\sigma_2^2}{1-\rho_2^2} \tag{2}$$

At the beginning of time period t, the actual inventory level at retailer (i) is $y_{t-1,i} - D_{t-1,i}$. Retailer (i) will order amount of product in order to reach the target inventory level $y_{t,i}$. Taking inventory sharing into consideration, the quantity for the demand at each retailer at the beginning of time period t can be determined as

$$q_{t,1} = y_{t,1} - (y_{t-1,1} - D_{t-1,1}) + \lambda q_{t,1}\\ = y_{t,1} - y_{t-1,1} + D_{t-1,1} + \lambda q_{t,1},$$

or

$$q_{t,1} = \frac{1}{1-\lambda}[y_{t,1} - y_{t-1,1} + D_{t-1,1}], \tag{3}$$

and

$$q_{t,2} = y_{t,2} - (y_{t-1,2} - D_{t-1,2}) - \lambda q_{t,1}\\ = y_{t,2} - y_{t-1,2} + D_{t-1,2} - \lambda q_{t,1}. \tag{4}$$

The target inventory $y_{t,i}$ at the beginning of period t is estimated from observed demand as

$$y_{t,1} = \widehat{D}_{t,1}^{L_1} + z_1 \widehat{\sigma}_{t,1}^{L_1},\\ y_{t,2} = \widehat{D}_{t,2}^{L2} + z_2 \widehat{\sigma}_{t,2}^{L_2}. \tag{5}$$

Both retailers use simple moving average technique to estimate $D_{t,i}^{Li}$ and $\sigma_{t,i}^{L_i}$ based on the information of the past p periods. We have

$$D_{t,i}^{L_i} = L_i\left(\frac{\sum_{j=1}^{p} D_{t-j,i}}{p}\right), \tag{6}$$

$$\sigma_{t,i}^{L_i} = C_{L_i,\rho}\sqrt{\frac{\sum_{j=1}^{p}\left(D_{t-j,i} - \widehat{D}_{t-j,i}\right)^2}{p}}. \tag{7}$$

where $D_{t-j,i} - \widehat{D}_{t-j,i}$ is the forecast error of the $(t-j)$th period at retailer (i) and $C_{L_i,\rho}$ is a constant function of L_i and ρ [26].The bullwhip effect can be calculated as $B_i = \frac{\mathrm{var}(q_{t,i})}{\mathrm{var}(D_i)}$.

3.4 Bullwhip effect quantify

Since the variance of $q_{t,i}$ and D_i is different in each retailer, the bullwhip effect at each supplier will be discussed separately. Given the equations of the order-up-to level, demand forecasting, and standard deviation of forecast error, $q_{t,1}$ can be expressed as follows:

$$\begin{aligned} q_{t,1} &= y_{t,1} - y_{t-1,1} + D_{t-1,1} + \lambda q_{t,1}\\ &= \frac{1}{1-\lambda}[y_{t,1} - y_{t-1,1} + D_{t-1,1}]\\ &= \frac{1}{1-\lambda}[(\widehat{D}_{t,1}^{L_1} + z_1\widehat{\sigma}_{t,1}^{L_1}) - (\widehat{D}_{t-1,1}^{L_1} + z_1\widehat{\sigma}_{t-1,1}^{L_1}) + D_{t-1,1}]\\ &= \frac{1}{1-\lambda}\left[\left(1+\frac{L_1}{p}\right)D_{t-1,1} - \left(\frac{L_1}{p}\right)D_{t-p-1,1} + z_1\left(\widehat{\sigma}_{t,1}^{L_1} - \widehat{\sigma}_{t-1,1}^{L_1}\right)\right]. \end{aligned} \tag{8}$$

Then, the variance of the order quantity $q_{t,1}$ at supplier (1) at time period t is as follows:

$$\begin{aligned} \mathrm{var}(q_{t,1}) &= \left(\frac{1}{1-\lambda}\right)^2 \mathrm{var}\left[\left(1+\frac{L_1}{p}\right)D_{t-1,1} - \left(\frac{L_1}{p}\right)D_{t-p-1,1}\right.\\ &\quad \left. + z_1(\widehat{\sigma}_{t,1}^{L_1} - \widehat{\sigma}_{t-1,1}^{L_1})\right]\\ &= \frac{\mathrm{var(D_1)}}{(1-\lambda)^2}\left[1 + \left(\frac{2L_1}{p} + \frac{2L_1^2}{p^2}\right)(1-\rho_1^p)\right]\\ &\quad + \frac{z_1^2}{(1-\lambda)^2}\mathrm{var}(\widehat{\sigma}_{t,1}^{L_1} - \widehat{\sigma}_{t-1,1}^{L_1}). \end{aligned} \tag{9}$$

For the variance of the order quantity $q_{t,2}$ at supplier (2) at time period t, we have

$$\begin{aligned} q_{t,2} &= y_{t,2} - y_{t-1,2} + D_{t-1,2} - \lambda q_{t,1}\\ &= (\widehat{D}_{t,2}^{L_2} + z_2\widehat{\sigma}_{t,2}^{L_2}) - (\widehat{D}_{t-1,2}^{L_2} + z_2\widehat{\sigma}_{t-1,2}^{L_2}) + D_{t-1,2} - \lambda q_{t,1}\\ &= \left(1+\frac{L_2}{p}\right)D_{t-1,2} - \left(\frac{L_2}{p}\right)D_{t-p-1,2} + z_2(\widehat{\sigma}_{t,2}^{L_2} - \widehat{\sigma}_{t-1,2}^{L_2}) - \lambda q_{t,1}. \end{aligned}$$

Then, the variance of the order quantity $q_{t,2}$ time period t is as follows:

$$\begin{aligned} &\mathrm{var}(q_{t,2})\\ &= \mathrm{var}\left[\left(1+\frac{L_2}{p}\right)D_{t-1,2} - \left(\frac{L_2}{p}\right)D_{t-p-1,2} + z_2\left(\widehat{\sigma}_{t,2}^{L_2} - \widehat{\sigma}_{t-1,2}^{L_2}\right)\right]\\ &\quad - \lambda^2\mathrm{var}(q_{t,1})\\ &= \left[\left(1 + \frac{2L_2}{p} + \frac{2L_2^2}{p^2}\right) - \left(\frac{2L_2}{p} + \frac{2L_2^2}{p^2}\right).\rho_2^p\right].\mathrm{var}(D_2)\\ &\quad + z_2^2\mathrm{var}\left(\widehat{\sigma}_{t,2}^{L_2} - \widehat{\sigma}_{t-1,2}^{L_2}\right) - \lambda^2\mathrm{var}(q_{t,1}). \end{aligned} \tag{10}$$

We denote B_1, B_2 are the bullwhip effect of supplier (1) and supplier (2) in period t, respectively. We have

$$\begin{aligned} B_1 &= \frac{\mathrm{var(q_{t,1})}}{\mathrm{var}(D_1)}\\ &= \frac{1}{(1-\lambda)^2}\left[1 + \left(\frac{2L_1}{p} + \frac{2L_1^2}{p^2}\right)(1-\rho^p)\right]\\ &\quad + \frac{z_1^2}{(1-\lambda)^2}\mathrm{var}(\widehat{\sigma}_{t,1}^{L_1} - \widehat{\sigma}_{t-1,1}^{L_1}). \end{aligned} \tag{11}$$

$$B_2 = \frac{\mathrm{var(q_{t,2})}}{\mathrm{var}(D_2)} = \left[1 + \left(\frac{2L_2}{p} + \frac{2L_2^2}{p^2}\right)(1-\rho^p)\right] + z_2^2 \mathrm{var}(\widehat{\sigma}_{t,2}^{L_2} - \widehat{\sigma}_{t-1,2}^{L_2}) - \frac{\lambda^2}{(1-\lambda)^2}\left[1 + \left(\frac{2L_1}{p} + \frac{2L_1^2}{p^2}\right)(1-\rho^p)\right]\frac{\mathrm{var(D_1)}}{\mathrm{var}(D_2)} - \frac{z_1^2\lambda^2}{(1-\lambda)^2}\frac{\mathrm{var}(\widehat{\sigma}_{\mathrm{t,2}}^{\mathrm{L_2}} - \widehat{\sigma}_{\mathrm{t-1,2}}^{\mathrm{L_2}})}{\mathrm{var}(D_2)}. \quad (12)$$

4 Numerical example and analysis

From (11) and (12), we can see that B_1 and B_2 are functions of the following parameters: p- the number of observation use in MA; L_1, L_2-the order lead time; ρ_1, ρ_2-first order autocorrelation coefficients of the autoregressive demand process at retailer (1) and retailer (2), respectively; λ-inventory sharing percentage. In this section, we will provide a numerical example to illustrate the impact of those parameters on the bullwhip effect on both retailers.

4.1 System description

Two trading companies A and B has center in North and South Vietnam, respectively. Company A imports steel from manufacturer (1) which has center in Italy while company B imports steel from manufacturer (2) which has center in Japan. Estimated order lead time of company A is 45 days, estimated order lead time of company B is 25 days. Both company A and B conduct forecasting and placing order two times per year (2 periods per year, each period has duration of 6 months). The order is placed at the beginning of each period. In Vietnam, construction is highly depending on the weather. In addition, the weather in North Vietnam and South Vietnam is quite different due to geometry position. To reduce risk of overstock, stock out as well increase customer service, both companies have signed an inventory sharing contract. The condition in the contract is revised and signed before every order period. The content of the contract state that in certain period, one company will agree to share λ percentage of its inventory to the other in case this company is in stock out state and other sharing conditions are satisfied. To enhance quality of forecast technique, both companies use 3 demand observation periods in the moving average forecast ($p = 3$). Demand observations used in forecast are corresponded to time of years. That means to forecast demand for spring period, only demand observations of spring period in the pass is applied and similar for autumn period. The autocorrelation coefficient of the autoregressive model of product each company is $\rho_1 = \rho_2 = 0.5$. In this paper, we will not focus in determine absolutely bullwhip effect in each manufacturer but compare the impact of inventory sharing policy on the bullwhip effect and the variation of bullwhip effect with some related parameters. For convenient purpose, bullwhip effect from each retailer (company) can be written as bellow:

$$B_1 = \frac{\mathrm{var(q_{t,1})}}{\mathrm{var}(D_1)} = \frac{1}{(1-\lambda)^2}\left[1 + \left(\frac{2L_1}{p} + \frac{2L_1^2}{p^2}\right)(1-\rho^p) + \theta_1\right],$$

$$B_2 = \left[1 + \left(\frac{2L_2}{p} + \frac{2L_2^2}{p^2}\right)(1-\rho^p) + \theta_2\right] - \frac{\lambda^2}{(1-\lambda)^2}[\psi],$$

where

$$\theta_1 = z_1^2 \mathrm{var}(\widehat{\sigma}_{t,1}^{L_1} - \widehat{\sigma}_{t-1,1}^{L_1}) > 0,$$

$$\theta_2 = z_2^2 \mathrm{var}(\widehat{\sigma}_{t,2}^{L_2} - \widehat{\sigma}_{t-1,2}^{L_2}) > 0,$$

$$\psi = 1 + \left(\frac{2L_1}{p} + \frac{2L_1^2}{p^2}\right)(1-\rho^p)\frac{\mathrm{var(D_1)}}{\mathrm{var}(D_2)} - z_1^2\frac{\mathrm{var}(\widehat{\sigma}_{\mathrm{t,2}}^{\mathrm{L_2}} - \widehat{\sigma}_{\mathrm{t-1,2}}^{\mathrm{L_2}})}{\mathrm{var}(D_2)} > 0.$$

4.2 Results and discussion

4.2.1 No inventory sharing ($\lambda = 0$)

If $\lambda = 0$, that is, there is no inventory sharing policy between the two companies, B_1 and B_2 are determined as follow:

$$B_1 = \left[1 + \left(\frac{2L_1}{p} + \frac{2L_1^2}{p^2}\right)(1-\rho^p)\right] + \theta_1,$$

$$B_2 = \left[1 + \left(\frac{2L_2}{p} + \frac{2L_2^2}{p^2}\right)(1-\rho^p)\right] + \theta_2.$$

In this case, we may consider the chain as two independent supply chains with single manufacturer, company, and customer. These results are identified with results of Li [31]. In Chen et al. [3], the authors pointed out the effects of common parameters in the bullwhip effect. The details are as follows:

- The bullwhip effect is a decreasing function of p, the number of observations use in MA.
- The bullwhip effect is an increasing function of L, the lead time.
- The bullwhip effect is a decreasing function of ρ when $\rho > 0$ and larger for odd values of p than for even values of p, when $\rho < 0$.

With the given information and relevant data, bullwhip effect in both manufacturer can be calculated as

$$B_1^0 = 420.875 + \theta_1, \quad (13)$$

$$B_2^0 = 136.9 + \theta_2. \quad (14)$$

4.2.2 Company A shares inventory to company B ($\lambda > 0$)

In case of $\lambda > 0$, we mean that company A will deliver amount of inventory, λq_1 to the warehouse of company B (if sharing conditions are satisfied) in the period t. Then, the bullwhip effect at manufacturer (1) and (2) are determined as:

$$B_1 = \frac{1}{(1-\lambda)^2} \times B_1^0 = \frac{420.875 + \theta_1}{(1-\lambda)^2} \quad (15)$$

$$B_2 = B_2^0 - \frac{\lambda^2}{(1-\lambda)^2}[\psi] = (136.9 + \theta_2) - \frac{\lambda^2}{(1-\lambda)^2}[\psi] \quad (16)$$

- From (13) and (15), it is clear that the bullwhip effect at manufacturer (1) in case of inventory sharing is higher than that in case of without sharing (since $\frac{1}{(1-\lambda)^2} > 1$), and the bullwhip effect is an increasing function of λ. That means, if amount of delivered inventory increase, the bullwhip effect at manufacturer (1) also increase.
- From (14) and (16), the bullwhip effect at manufacturer (2) in case of sharing is smaller than that in case of without sharing. Furthermore, the bullwhip effect quantity is a deceasing function of λ which means that higher transhipment company B receive from company A, smaller the bullwhip effect at manufacture (1). The reason is that the variation of customer demand at supplier is a result of changing customer information through middle parties such as retailer, wholesaler. Because those parties always desire to keep desirable service level, the order quantity placing at suppliers are usually higher than actual demands. When one retailer is expected to receive a certain amount of goods from another, the order quantity this retailer place at the supplier will reduce and closer to the actual demands. That leads to the bullwhip effect reduces.

The variation of bullwhip effect with inventory sharing percentage in each manufacturer is given in Figs. 4 and 5. Herein, we assign $\theta_1 = \theta_2 = \psi = 1$.

Because the role of each party in the chain is equivalent, we can refer that in case of company B agree to share its inventory to company A ($\lambda < 0$) the result is that when $|\lambda|$ increases, the bullwhip effect at manufacturer (1) decreases, while the bullwhip effect at manufacturer (2) increases.

5 Conclusions and recommendations

In this paper, we considered the impact of inventory sharing on decentralized warehouses system on the bullwhip effect by comparing the bullwhip effects at both suppliers in case of no inventory sharing policy with that in case of using inventory sharing policy. The studying supply chain system includes two suppliers, retailers, and markets.

By constructing the bullwhip effect at each supplier for different study situations, we found that inventory sharing policy has significant impact on the bullwhip effect in each supplier. The variation of the bullwhip effect in each

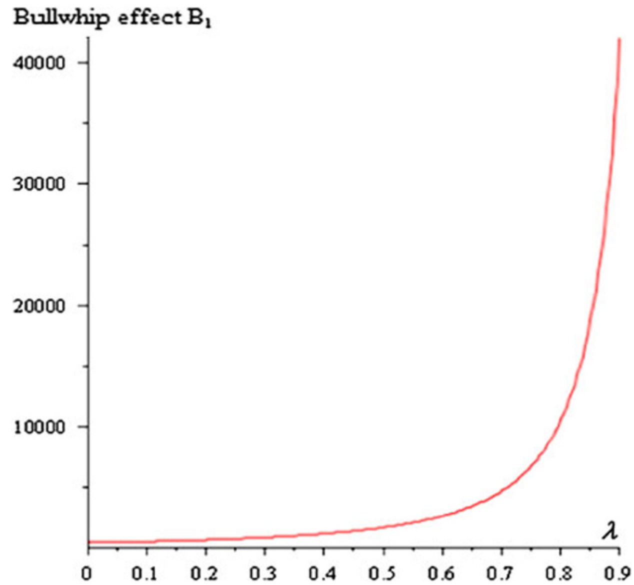

Fig. 4 Variation of the bullwhip effect in manufacturer (1) with inventory sharing percentage

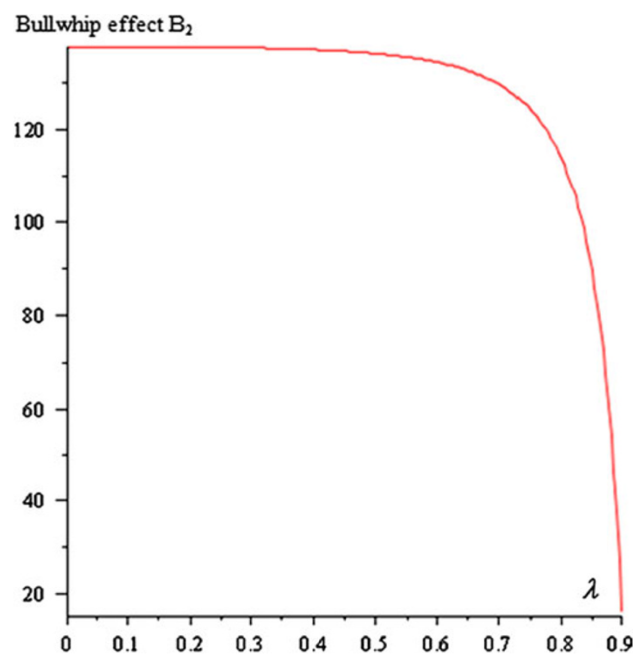

Fig. 5 Variation of the bullwhip effect in manufacturer (2) with inventory sharing percentage

supplier depends on the destination of transferring inventory. In details, the bullwhip effect at one supplier will increase if its retailer is the one that received transferring inventory. In addition, the higher amount of receiving inventory of the retailer, the bigger the bullwhip effect in the corresponding supplier. In other words, inventory sharing reduces the bullwhip effect at this supplier but also increases the bullwhip effect at the other supplier.

According to the finding of this paper, supply chain managers are helpful in forecasting amount of goods that need to supply for their retailers. Also supply chain managers can adjust inventory sharing policy in order to trade-off the bullwhip effect for both suppliers. In many cases, inventory sharing profits for the whole supply system, therefore, determining the impact of inventory sharing on the bullwhip effect will further support for the development of inventory sharing models in supply chain.

This paper can be extended through three directions. One is that we can study the impact of inventory sharing on the bullwhip effect in a supply chain with only one supplier and two or multi-retailers. With inventory sharing, total product quantity from retailers may differ from that in case of without inventory sharing. Another side direction would be extending the model to multi-stage supply chain, and inventory sharing is applied in different levels in the chain. For the third direction, we can study this model with multi-type products.

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Appendix

1. The derivation process of $E(D_{t,i})$ and $\mathrm{var}(D_{t,i})$When the autoregressive demand process is stationary, we have

$$E(D_{t,i}) = E(D_{t-1,i}) = E(D_{t-2,i}) = \ldots = E(D_i),$$

and

$$\mathrm{var}(D_{t,i}) = \mathrm{var}(D_{t-1,i}) = \mathrm{var}(D_{t-2,i}) = \ldots = \mathrm{var}(D_i),$$

where

$$\begin{aligned}
&D_{t,1} = \mu_1 + \rho_1 D_{t-1,1} + \varepsilon_{t,1},\\
&E(D_{t,1}) = E(\mu_1) + \rho_1 E(D_{t-1,1}) + E(\varepsilon_{t,1})\\
&E(D_1) = \mu_1 + \rho_1 E(D_1) + 0\\
&\Rightarrow E(D_1) = \frac{\mu_1}{1-\rho_1}.\\
&\mathrm{var}(D_{t,1}) = \mathrm{var}(\mu_1) + \rho_1^2 \mathrm{var}(D_{t-1,1}) + \mathrm{var}(\varepsilon_{t,1})\\
&\mathrm{var}(D_1) = 0 + \rho_1^2 \mathrm{var}(D_1) + \sigma_1^2\\
&\Rightarrow \mathrm{var}(D_1) = \frac{\sigma_1^2}{1-\rho_1^2}.
\end{aligned}$$

Similarly, we have

$$E(D_2) = \frac{\mu_2}{1-\rho_2},$$
$$\mathrm{var}(D_1) = \frac{\sigma_2^2}{1-\rho_2^2}.$$

2. The derivation process of the further equation of $q_{t,1}$.

$$\begin{aligned}
q_{t,1} &= y_{t,1} - y_{t-1,1} + D_{t-1,1} + \lambda q_{t,1},\\
\Rightarrow q_{t,1} &= \frac{1}{1-\lambda}\left[y_{t,1} - y_{t-1,1} + D_{t-1,1}\right]\\
&= \frac{1}{1-\lambda}\left[\left(\widehat{D}_{t,1}^{L_1} + z_1\widehat{\sigma}_{t,1}^{L_1}\right) - \left(\widehat{D}_{t-1,1}^{L_1} + z_1\widehat{\sigma}_{t-1,1}^{L_1}\right) + D_{t-1,1}\right]\\
&= \frac{1}{1-\lambda}\left[\left(\widehat{D}_{t,1}^{L_1} - \widehat{D}_{t-1,1}^{L_1}\right) + D_{t-1,1} - z_1\left(\widehat{\sigma}_{t,1}^{L_1} - \widehat{\sigma}_{t-1,1}^{L_1}\right)\right]\\
&= \frac{L}{p(1-\lambda)}\left(\sum_{i=1}^{p} D_{t-i,1} - \sum_{i=1}^{p} D_{t-1-i,1}\right)\\
&\quad + \frac{D_{t-1,1}}{1-\lambda} + \frac{z_1}{1-\lambda}\left(\widehat{\sigma}_{t,1}^{L_1} - \widehat{\sigma}_{t-1,1}^{L_1}\right)\\
&= \frac{L}{p(1-\lambda)}\left(D_{t-1,1} - D_{t-p-1,1}\right) + \frac{D_{t-1,1}}{1-\lambda}\\
&\quad + \frac{z_1}{1-\lambda}\left(\widehat{\sigma}_{t,1}^{L_1} - \widehat{\sigma}_{t-1,1}^{L_1}\right)\\
&= \frac{1}{1-\lambda}\left(1 + \frac{L_1}{p}\right)D_{t-1,1} - \frac{L_1}{p(1-\lambda)}D_{t-p-1,1}\\
&\quad + \frac{z_1}{1-\lambda}\left(\widehat{\sigma}_{t,1}^{L_1} - \widehat{\sigma}_{t-1,1}^{L_1}\right),
\end{aligned}$$

$$\mathrm{var}(q_{t,1}) = \mathrm{var}\begin{pmatrix} \frac{1}{1-\lambda}\left(1 + \frac{L_1}{p}\right)D_{t-1,1} - \frac{L_1}{p(1-\lambda)}D_{t-p-1,1} \\ + \frac{z_1}{1-\lambda}\left(\widehat{\sigma}_{t,1}^{L_1} - \widehat{\sigma}_{t-1,1}^{L_1}\right) \end{pmatrix}$$

$$= \frac{1}{(1-\lambda)^2}\begin{pmatrix} \left(1 + \frac{L_1}{p}\right)^2 \mathrm{var}(D_{t-1,1}) \\ - 2\left(\frac{L_1}{p}\right)\left(1 + \frac{L_1}{p}\right)\mathrm{cov}(D_{t-1,1}, D_{t-p-1,1}) \\ + \left(\frac{L_1}{p}\right)^2 \mathrm{var}(D_{t-p-1,1}) + z_1^2 \mathrm{var}(\widehat{\sigma}_{t,1}^{L_1} - \widehat{\sigma}_{t-1,1}^{L_1}) \\ + 2z_1\left(1 + \frac{2L_1}{p}\right)\mathrm{cov}(D_{t-1,1}, \widehat{\sigma}_{t,1}^{L_1}) \end{pmatrix}$$

$$= \frac{1}{(1-\lambda)^2}\begin{pmatrix} \left(1 + \frac{2L_1}{p} + \left(\frac{2L_1}{p}\right)^2\right)\mathrm{var}(D_1) \\ - \left(\frac{2L_1}{p} + \frac{2L_1^2}{p}\right)\mathrm{cov}(D_{t-1,1}, D_{t-p-1,1}) \\ + z_1^2 \mathrm{var}(\widehat{\sigma}_{t,1}^{L_1} - \widehat{\sigma}_{t-1,1}^{L_1}) \\ + 2z_1\left(1 + \frac{2L_1}{p}\right)\mathrm{cov}(D_{t-1,1}, \widehat{\sigma}_{t,1}^{L_1}) \end{pmatrix}.$$

Now we will determine $\mathrm{cov}(D_{t-1,1}, D_{t-p-1,1})$ and $\mathrm{cov}(D_{t-1,1}, \widehat{\sigma}_{t,1}^{L_1})$ We have

$$\begin{aligned}&\operatorname{cov}(D_{t-1,1}, D_{t-p-1,1})\\&= \operatorname{cov}\big((\mu_1 + \rho_1 D_{t-2,1} + \varepsilon_{t,1}), D_{t-p-1,1}\big)\\&= \operatorname{cov}\big(\mu_1, D_{t-p-1,1}\big) + \rho_1 \operatorname{cov}\big(D_{t-2,1}, D_{t-p-1,1}\big)\\&+ \operatorname{cov}\big(\varepsilon_{t,1}, D_{t-p-1,1}\big).\end{aligned}$$

$$\begin{aligned}&(\text{Since} \operatorname{cov}\big(\mu_1, D_{t-p-1,1}\big) = 0 \text{ and} \operatorname{cov}\big(\varepsilon_{t,1}, D_{t-p-1,1}\big) = 0),\\&\operatorname{cov}(D_{t-1,1}, D_{t-p-1,1}) = \rho_1 \operatorname{cov}\big(D_{t-2,1}, D_{t-p-1,1}\big)\\&\qquad \ldots\\&\qquad = \rho_1^p \operatorname{cov}\big(D_{t-p,1}, D_{t-p-1,1}\big)\\&\qquad = \rho_1^p \operatorname{var}(D_1).\end{aligned}$$

We assume that forecasting customer demands by retailers are random variables of the form as $D_t = \mu + \rho D_{t-1} + \varepsilon_t$, and the error terms $\varepsilon_t$are identically independent distribution with mean 0 and variance σ^2. Let the estimate of the standard deviation of forecast error of the lead time demand be

$$\widehat{\sigma}_t^L = C_{L,p} \sqrt{\frac{\sum_{i=j}^{p} (D_{t-j} - \widehat{D}_{t-j})^2}{p}}.$$

Applying the result proved in Ryan [21], we have

$$\operatorname{cov}(D_{t-j}, \widehat{\sigma}_t^L) = 0, \quad \forall i = 1, 2, \ldots, p.$$

Hence,

$$\begin{aligned}&\operatorname{var}(q_{t,1})\\&= \frac{1}{(1-\lambda)^2}\left[\begin{array}{l}\left(1 + \frac{2L_1}{p} + 2\left(\frac{L_1}{p}\right)^2\right)\operatorname{var}(D_1)\\ -\left(\frac{2L_1}{p} + \frac{2L_1^2}{p}\right)\operatorname{cov}(D_{t-1,1}, D_{t-p-1,1})\\ + z_1^2 \operatorname{var}(\widehat{\sigma}_{t,1}^{L_1} - \widehat{\sigma}_{t-1,1}^{L_1}) + 2z_1\left(1 + \frac{2L_1}{p}\right)\operatorname{cov}(D_{t-1,1}, \widehat{\sigma}_{t,1}^{L_1})\end{array}\right]\\&= \frac{1}{(1-\lambda)^2}\left[\begin{array}{l}\left(1 + \frac{2L_1}{p} + 2\left(\frac{L_1}{p}\right)^2\right)\operatorname{var}(D_1) - \left(\frac{2L_1}{p} + \frac{2L_1^2}{p}\right)\rho_1^p \operatorname{var}(D_1)\\ + z_1^2 \operatorname{var}(\widehat{\sigma}_{t,1}^{L_1} - \widehat{\sigma}_{t-1,1}^{L_1})\end{array}\right]\\&= \frac{\operatorname{var}(D_1)}{(1-\lambda)^2}\left[1 + \left(\frac{2L_1}{p} + \frac{2L_1^2}{p}\right)(1 - \rho_1^p)\right] + \frac{z_1^2 \operatorname{var}(\widehat{\sigma}_{t,1}^{L_1} - \widehat{\sigma}_{t-1,1}^{L_1})}{(1-\lambda)^2}.\end{aligned}$$

3. The derivation process of the further expression of $q_{t,2}$:

$$\begin{aligned}q_{t,2} &= \left(1 + \frac{L_2}{p}\right)D_{t-1,2} - \left(\frac{L_2}{p}\right)D_{t-p-1,2}\\&\quad + z_2(\widehat{\sigma}_{t,2}^{L_2} - \widehat{\sigma}_{t-1,2}^{L_2}) - \lambda q_{t,1}, \Rightarrow \operatorname{var}(q_{t,2})\\&= \operatorname{var}\left[\left(1 + \frac{L_2}{p}\right)D_{t-1,2} - \left(\frac{L_2}{p}\right)D_{t-p-1,2} + z_2(\widehat{\sigma}_{t,2}^{L_2} - \widehat{\sigma}_{t-1,2}^{L_2})\right]\\&\quad - \lambda^2 \operatorname{var}(q_{t,1}) = \left(1 + \frac{L_2}{p}\right)^2 \operatorname{var}(D_{t-1,2})\\&\quad - 2\left(\frac{L_2}{p}\right)\left(1 + \frac{L_2}{p}\right)\operatorname{cov}(D_{t-1,2}, D_{t-p-1,2})\\&\quad + \left(\frac{L_2}{p}\right)^2 \operatorname{var}(D_{t-p-1,2}) + z_2^2 \operatorname{var}(\widehat{\sigma}_{t,2}^{L_2} - \widehat{\sigma}_{t-1,2}^{L_2})\\&\quad + 2z_2\left(1 + 2\frac{L_2}{p}\right)\operatorname{cov}(D_{t-1,2}, \widehat{\sigma}_{t,2}^{L_2}) - \lambda^2 \operatorname{var}(q_{t,1})\\&= \left(1 + 2\frac{L_2}{p} + 2\left(\frac{L_2}{p}\right)^2\right)\operatorname{var}(D_2)\\&\quad - \left(\frac{2L_2}{p} + 2(\frac{L_2}{p})^2\right)\operatorname{cov}(D_{t-1,2}, D_{t-p-1,2})\\&\quad + z_2^2 \operatorname{var}(\widehat{\sigma}_{t,2}^{L_2} - \widehat{\sigma}_{t-1,2}^{L_2}) + 2z_2\left(1 + \frac{2L_2}{p}\right)\operatorname{cov}(D_{t-1,2}, \widehat{\sigma}_{t,2}^{L_2})\\&\quad - \lambda^2 \operatorname{var}(q_{t,1}).\end{aligned}$$

Furthermore, we have $\operatorname{cov}(D_{t-1,2}, \widehat{\sigma}_{t,2}^{L_2}) = 0$ and

$$\begin{aligned}&\operatorname{cov}(D_{t-1,2}, D_{t-p-1,2})\\&= \operatorname{cov}(\mu_2 + \rho_2 D_{t-2,2} + \varepsilon_{t-1,2}, D_{t-p-1,2})\\&= \operatorname{cov}(\mu_2, D_{t-p-1,2}) + \rho_2 \operatorname{cov}(D_{t-2,2}, D_{t-p-1,2})\\&\quad + \operatorname{cov}(\varepsilon_{t-1,2}, D_{t-p-1,2})\\&= \rho_2 \operatorname{cov}(D_{t-1,2}, D_{t-p-1,2})\\&\ldots\\&= \rho_2^p \operatorname{cov}(D_{t-p-1,2}, D_{t-p-1,2})\\&= \rho_2^p \operatorname{var}(D_2).\end{aligned}$$

$(\text{Note that} \operatorname{cov}(\mu_2, D_{t-p-1,2}) = 0 \text{ and } \operatorname{cov}(\varepsilon_{t-1,2}, D_{t-p-1,2}) = 0)$

Hence,

$$\begin{aligned}\operatorname{var}(q_{t,2}) &= \left(1 + 2\frac{L_2}{p} + 2\left(\frac{L_2}{p}\right)^2\right)\operatorname{var}(D_2)\\&\quad - \left(\frac{2L_2}{p} + 2\left(\frac{L_2}{p}\right)^2\right)\rho_2^2 \operatorname{var}(D_2)\\&\quad + z_2^2 \operatorname{var}(\widehat{\sigma}_{t,2}^{L_2} - \widehat{\sigma}_{t-1,2}^{L_2}) - \lambda^2 \operatorname{var}(q_{t,1})\\&= \operatorname{var}(D_2)\left[1 + \left(2\frac{L_2}{p} + 2\left(\frac{L_2}{p}\right)^2\right)(1 - \rho_2^2)\right]\\&\quad + z_2^2 \operatorname{var}(\widehat{\sigma}_{t,2}^{L_2} - \widehat{\sigma}_{t-1,2}^{L_2}) - \lambda^2 \operatorname{var}(q_{t,1}).\end{aligned}$$

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